

The Resolvent Method

with a first application on building a bridge between
Schramm-Loewner Evolutions and Random Matrix
Theory and a frontier in Machine Learning: ML for
Singular Dynamics

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partially based on joint works with A. Campbell and K. Luh; N. Shrotri

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Let me first introduce myself

- Random Matrix Theory
- Schramm-Loewner Evolutions (and connections with Rough Paths)
- (Long-Range) Statistical Mechanics models
- Cover time of Brownian motion on the Brownian continuum random tree (Aldous' 1991 Conjecture, recently in AOP)
- Probabilistic Number Theory

More on www.margarintvlad.com

Thank you to all the speakers and participants for making this session possible!

Planar Statistical Physics. What is the scaling limit?

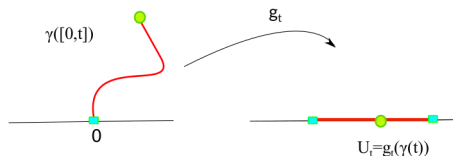
- A sample of a planar Loop-Erased Random Walk:



Figure: Credit to E. Peltola, A. Karrila, K. Kytola.

Conformal maps and the Loewner equation

- In general, for a non-self crossing curve $\gamma(t) : [0, \infty) \rightarrow \bar{\mathbb{H}}$ with $\gamma(0) = 0$ and $\gamma(\infty) = \infty$, we consider the simply connected domain $\mathbb{H} \setminus \gamma([0, t])$.



- Using the Riemann Mapping Theorem for the simply connected domain $\mathbb{H} \setminus \gamma([0, t])$, we have a three parameter family of conformal maps $g_t : \mathbb{H} \setminus \gamma([0, t]) \rightarrow \mathbb{H}$.
- Loewner Equation encodes the dependence between the evolution of the maps g_t when the curve $\gamma([0, t])$ grows.
- The maps satisfy the Loewner Differential Equation

$$\partial_t g_t(z) = \frac{2}{g_t(z) - U_t}, \quad g_0(z) = z.$$

Definition of SLE

Definition

Let B_t be a standard real Brownian motion starting from 0. The chordal SLE(κ) is defined as the law on curves induced by the solution to the following ordinary differential equation

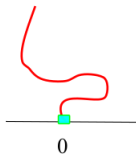
$$\partial_t g_t(z) = \frac{2}{g_t(z) - \sqrt{\kappa} B_t}, \quad g_0(z) = z.$$

- SLE trace: $\gamma(t) = \lim_{y \rightarrow 0+} g_t^{-1}(\sqrt{\kappa} B_t + iy)$.
[Rohde-Schramm ($\kappa \neq 8$), Lawler-Schramm-Werner($\kappa = 8$)]

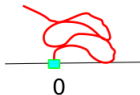
SLE phase transitions

- It is proved that there are two phase transitions when κ varies between 0 and ∞ .

Up to $\kappa = 4$



$4 < \kappa < 8$



$\kappa = 8$ and bigger



Multiple SLE model

- **Brownian Motion** $\rightarrow$ **Dyson Brownian Motion**(DBM), $\beta = 8/\kappa$.

$$d\lambda_t^i = \frac{\sqrt{2}}{\sqrt{N\beta}} dB_t^i + \frac{1}{N} \sum_{j \neq i} \frac{dt}{\lambda_t^j - \lambda_t^i}, i = 1, \dots, N.$$

- **Multiple SLE** simultaneous growth

$$\partial_t g_t^N(z) = \frac{1}{N} \sum_{j=1}^N \frac{2}{g_t^N(z) - \lambda_t^j}.$$

Theorem (Campbell-Luh-M, **featured article in RMTA 2025.**)

Let $\beta = 1$ or $\beta = 2$, and let K_T^N be the multiple SLE hull with N drivers at time $T > 0$. Then, for any $\epsilon > 0$ and compact $G \subset \mathbb{H} \setminus \cup_N K_T^N$, for the multiple SLE maps with N -drivers, we have that

$$\sup_{t \in [0, T], z \in G} |g_t^N(z) - g_t^\infty(z)| = O_{G, T} \left(\frac{1}{N^{1/3-\epsilon}} \right),$$

with overwhelming probability.

- E holds with o.p. if, for every $p > 0$, $\mathbb{P}(E) \geq 1 - O(N^{-p})$.
- Uses modern RMT techniques such as Stieltjes transforms, self-consistent equations, etc.

Idea of the proof of the result

First, Del Monaco and Schleissinger:

$$\frac{\partial}{\partial t} g_t^\infty(z) = M_t^\infty(g_t(z)), \quad g_0(z) = z,$$

where M_t^∞ is a solution to the complex Burgers equation

$$\begin{cases} \frac{\partial M_t^\infty(z)}{\partial t} = -2M_t^\infty(z) \frac{\partial M_t^\infty(z)}{\partial z}, & t > 0 \\ M_0^\infty(z) = \int_{\mathbb{R}} \frac{2}{z-x} d\mu_0(x) \end{cases}$$

- Let $M_t^N(\bullet) = \frac{1}{N} \sum_{j=1}^N \frac{2}{\bullet - \lambda_t^j}$.
- Let us consider the time interval $[0, 1]$ and a uniform partition with $t_k = \frac{k}{n}, k = 0, 1, \dots, n$. Let $t \in (t_1, t_2)$.
- The proof is based on controlling
$$\sup_{\bullet \in G} |M_t^\infty(\bullet) - M_t^N(\bullet)| \leq \sup_{\bullet \in G} |M_t^\infty(\bullet) - M_{t_1}^\infty(\bullet)| + \sup_{\bullet \in G} |M_{t_1}^\infty(\bullet) - M_{t_1}^N(\bullet)| + \sup_{\bullet \in G} |M_{t_1}^N(\bullet) - M_t^N(\bullet)|, \text{ for } G \subset \mathbb{H}.$$

Idea of the proof of the result

- Let $\beta = 1$. We have that $M_t^N(\bullet) = \frac{-2}{N} \text{tr}(A_t - \bullet \cdot I)^{-1}$, where $A_t = \sqrt{t}A$, with A a GOE.
- For M_t^∞ , we have $M_t^\infty(\bullet) - S^N(z - 2tM_t^\infty(\bullet)) = 0$, with $S^N(\bullet) = -\frac{2}{N} \text{tr} Q(\bullet)$, where $Q(\bullet)$ is a certain resolvent matrix.
- By a union bound, we have that, for any $\epsilon > 0$

$$\begin{aligned} \mathbb{P} \left(\bigcup_{t_k} \left| M_{t_k}^\infty(\bullet) - M_{t_k}^N(\bullet) \right| = \Omega \left(\frac{C}{N^{1/3-\epsilon}} \right) \right) \\ \leq \sum_{k=1}^n \mathbb{P} \left(\left| M_{t_k}^\infty(\bullet) - M_{t_k}^N(\bullet) \right| = \Omega \left(\frac{C}{N^{1/3-\epsilon}} \right) \right) \\ \leq ne^{-CN}. \end{aligned}$$

- Note $g = \Omega(f) \leftrightarrow f = O(g)$.

Summary of the toolbox between RMT and SLE

$$\partial_t g_t(z) = \frac{1}{N} \sum_{j=1}^N \frac{2}{g_t(z) - \lambda_t^j}.$$

- Rewrite RHS of Multiple Loewner equation as $M_t^N(\bullet) = \frac{1}{N} \sum_{j=1}^N \frac{2}{\bullet - \lambda_t^j}$.
- Recognize **Stieltjes transform** for a given choice of measure.
- **Let $\beta = 1$. We have that $M_t^N(\bullet) = \frac{-2}{N} \text{tr}(A_t - \bullet \cdot I)^{-1}$, where $A_t = \sqrt{t}A$, with A a GOE**
- RHS of Loewner can be recovered as a Random Matrix Theory object.
- **The critical parameters $\kappa = 4$ and $\kappa = 8$ in the SLE theory correspond to the 'nice' $\beta = 2$ and $\beta = 1$ parameters in the Random Matrix Theory.**

The result presented today is a first application of this toolbox.

Advantages of using this method

- Multiple SLE for $\beta = 1$ or $\beta = 2$, i.e. **critical** $\kappa = 8$ or $\kappa = 4$:

$$\partial_t g_t(z) = \frac{1}{N} \sum_{j=1}^N \frac{2}{g_t(z) - \lambda_t^j}, \text{ with } \mathbf{singular\ driver\ dynamics}$$

$$d\lambda_t^i = \frac{\sqrt{2}}{\sqrt{N\beta}} dB_t^i + \frac{1}{N} \sum_{j \neq i} \frac{dt}{\lambda_t^j - \lambda_t^i}, i = 1, \dots, N. \quad (1)$$

or

$$d\lambda_t^i = \frac{\sqrt{2}}{\sqrt{N\beta}} dB_t^i + \frac{1}{N} \sum_{j \neq i} \frac{dt}{\lambda_t^j - \lambda_t^i} - \frac{\lambda_t^i}{2} dt, i = 1, \dots, N. \quad (2)$$

- RHS of Multiple SLE in the first item above: $M_t^N(\bullet) = \frac{1}{N} \sum_{j=1}^N \frac{2}{\bullet - \lambda_t^j}$.
- Alterative study** of the first item **via resolvent**(nice analytical object)

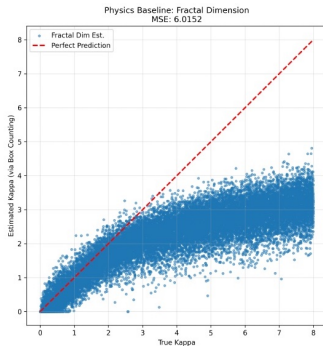
$$M_t^N(\bullet) = \frac{-2}{N} \text{tr} (A_t - \bullet \cdot I)^{-1},$$

for $A_t = \sqrt{t}A$, with A GOE or GUE (for (1)), or A_t satisfying a nice matrix dynamics (for (2))

$$dA_t = -\frac{1}{2} A_t dt + \frac{1}{\sqrt{N}} dB_t$$

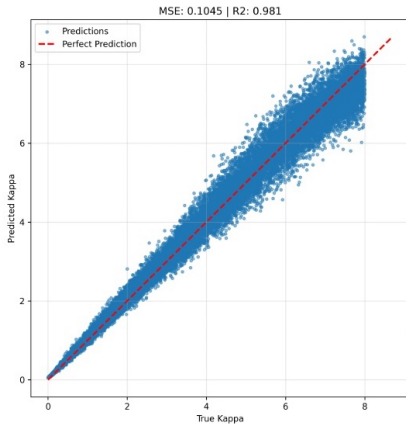
ML for Singular Dynamics: Neural Networks and SLE

- With N. Shrotri, we trained NN.
- First, prediction via Fractal Dimension $D(\kappa) = 1 + \kappa/8$:



ML for Singular Dynamics: Neural Networks and SLE

- Second, prediction via NN:



- This work contains other parameters estimations in Loewner theory.
- This work appeared this year in *Neural Computing and Applications* by Springer and is, to the best of my knowledge, the first work on the interface of ML and SLE.

Thank you very much for your
attention!