Perturbations of Multiple SLE with two non-colliding Dyson BMs

In just 2 slides

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- Consider the Loewner Differential Equation

$$\partial_t g_t(z) = \frac{2}{g_t(z) - U_t}, \qquad g_0(z) = z\,.$$

- SLE: $U_t = \sqrt{\kappa} B_t$.
- Our work: **Brownian Motion** $\rightarrow$ **Dyson Brownian Motion**(DBM), $\beta = 8/\kappa$.

$$d\lambda_t^i = \frac{\sqrt{2}}{\sqrt{N\beta}} dB_t^i + \frac{1}{N} \sum_{j \neq i} \frac{dt}{\lambda_t^j - \lambda_t^i}, i = 1, \cdots, N.$$

We obtain a general N estimate in Carathéodory sense and for $N = 2$:

Theorem (Chen-M.)

For $N = 2$ drivers, perturbations of initial value/ parameter $\kappa \in (0, 4]$, convergence a.s /with high probability in the Carathéodory sense.

- For $\kappa, \kappa^* \in (0, 4]$, we consider

$$\partial_t g_t(z) = \frac{1}{g_t(z) - \lambda_t^1} + \frac{1}{g_t(z) - \lambda_t^2}$$

and

$$\partial_t g_t^*(z) = \frac{1}{g_t^*(z) - \lambda_t^{1*}} + \frac{1}{g_t^*(z) - \lambda_t^{2*}}.$$

- The (rescaled) gaps between the drivers are **Bessel processes** of dimensions $d = 1 + \frac{8}{\kappa}$, and $d^* = 1 + \frac{8}{\kappa^*}$.
- One can show that

$$\mathbb{P}\left(\|g_t(z) - g_t^*(z)\|_{[0,T]\times G} > \varphi\left(\kappa^* - \kappa\right)\right) < \zeta\left(\kappa^* - \kappa\right),$$

for some explicit functions $\varphi(\cdot)$ and $\zeta(\cdot)$ such that $\lim_{x\to 0} \varphi(x) = 0$, and $\lim_{x\to 0} \zeta(x) = 0$.

- **What can we learn?** We obtain a method to study perturbations of these models for the case $N = 2$ in which we have access to the statistics of the gap (Bessel Process).