On Loewner chains driven by semimartingales and complex Bessel-type SDEs

In just 2 slides

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https://margarintvlad.com/

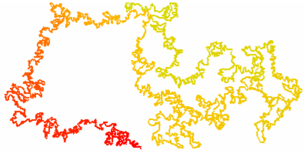

Figure: SLE$_\kappa$, for some $\kappa \in (4, 8)$. Credit: M. Raad

- The forward and backward Loewner Differential Equation

$$\dot{g}_t(z) = \frac{2}{g_t(z) - U_t}, \qquad \dot{h}_t(z) = \frac{-2}{h_t(z) - W_t}.$$

- For $U_t = \sqrt{\kappa} B_t$, SLE curves $\gamma(t) := \lim_{y \to 0+} g_t^{-1}(iy + \sqrt{\kappa} B_t)$.
- For $W_t = \sqrt{\kappa}(B_T - B_{T-t}) = \sqrt{\kappa}\tilde{B}_t$, via $h_t(z) - \sqrt{\kappa}\tilde{B}_t = Z_t$, we obtain

$$dZ_t = \frac{-2}{Z_t} dt - \sqrt{\kappa} d\tilde{B}_t.$$

- We **extend** this picture and we consider U_t and W_t to be **continuous semi-martingale** processes $S_t = M_t + A_t$, where M_t is a local martingale and A_t is a bounded variation process (+ conditions).

Theorem (M., Shekhar, Yuan)

If U is a semimartingale satisfying some conditions, then the Loewner chain with the driver W given by $W_t = U_T - U_{T-t}, t \in [0, T]$, is almost surely generated by a curve γ. Furthermore, under some extra conditions, γ is almost surely simple and $\gamma_t \in \mathbb{H}$ for $t \in (0, T]$.

- We obtain also a result for the case when W_t is a semi-martingale.
- Also, we obtain, the **existence and uniqueness of strong solutions** of a **Complex-Valued Bessel process** starting form the origin.
- **Difficulty:** Control $\sup_t \left|(g_t^{-1})'(iy + W_t)\right| \lesssim y^{-\theta}$, for some $\theta < 1$.
- **What can we learn?** We managed to extend the previous research to **Loewner chains driven by semi-martingales** and obtain **existence and uniqueness result for complex-valued Bessel processes** that extends the previous works in the literature for the real-valued ones.