Continuity in κ of the SLE_κ welding

In just 2 slides

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- **Coupled Bessel family:** For dimension $\delta \leq 0$, consider Bessel processes $\{Z_t(x)\}_{t\geq 0}$ started at $x \neq 0$, *all driven by the same Brownian motion* B_t, solving

$$dZ_t \ = \ dB_t + \frac{\delta - 1}{2}\frac{1}{Z_t}\,dt.$$

- Define the **zero-hitting time**

$$\zeta_\delta^x \ = \ \inf\{t \geq 0 : \ Z_t(x) = 0\}.$$

Theorem (Beliaev, M., Shekhar)

Almost surely, the map

$$(x, \delta) \longmapsto \zeta_\delta^x$$

is **jointly continuous** *for all* $x \in \mathbb{R}$ *and all* $\delta \leq 0$.

- Let us link this with the reverse Loewner dynamics

$$\partial_t h_t(z) = -\frac{2}{h_t(z) - \sqrt{\kappa}B_t}, \qquad h_0(z) = z.$$

- **Bridge to reverse Loewner dynamics:** For the reverse Loewner flow $h_t(x)$, with parameter $\delta(\kappa) = 1 - \frac{4}{\kappa}$, there is a scaling identity

$$\frac{h(s,t,\sqrt{\kappa}\,x)}{\sqrt{\kappa}} = Z_\delta(s,t,x),$$

  which identifies Loewner hitting times $T_\kappa(x)$ with Bessel hitting times ζ_δ^x.
- As the welding is studied through hitting times, the main result can be used to obtain the a.s. continuity in κ for $\kappa \in [0,4)$ of the welding.
- Prove that a.s. $x \mapsto T_\kappa(x)$ is **strictly monotone, continuous, and bijective**, and that $\kappa \mapsto T_\kappa(\cdot)$ is **continuous**.
- **Difficulty**: Hitting times control for a **singular** dynamics, and more.
- **What can we learn?** This joint continuity result is giving information on an important quantity about Bessel processes (which appear in other areas of research) and its applications in SLE give us a tool to prove the a.s. continuity of the welding homeorphism.